Progressive Commercial Arithmetic.

PART ONE.

THE
Progressive Commercial Arithmetic

FOR

COMMERCIAL SCHOOLS, HIGH SCHOOLS
AND ACADEMIES,

BY

SAMUEL H. GOODYEAR, A. M.
AND
WALLACE H. WHIGAM, LL. M.,

SIXTH EDITION

CEDAR RAPIDS, IOWA.
GOODYEAR-MARSHALL PUBLISHING CO.
1901.

Copyright by
Goodyear-Marshall Publishing Co.
1901.

PREFACE TO THE SIXTH EDITION.

The original plan of the Progressive Commercial Arithmetic contemplated a work for advanced commercial and other classes.

The first five editions began with Part II of the present edition, omitting the elementary work included under the fundamental topics, Notation, Numeration, Addition, Subtraction, Multiplication and Division.

But a demand has arisen for a more complete text for use in Normal and other classes, where a general and full discussion of the elementary processes is desirable.

To meet this demand the present edition has been prepared. It includes the work hitherto published, adding a full preliminary treatment of the fundamental topics, including a large number of drill problems and exercises, both oral and written.

It is believed that the practical and original character of the matter in Part I will promptly appeal to the progressive teacher, and that the book in its present form will better meet the needs of all schools that have classes in Commercial Arithmetic.

The study of Part I may be omitted, if, on account of limited time, or for any other reason, it is found desirable to commence with the more advanced topics.

PART I.

ARITHMETIC.

INTRODUCTION.

The idea of *number* is present in every sound human mind. Perhaps there is no better way of defining number than to say that it is the idea suggested by the question, "how many?"

NOTE.—The student, in his thinking, should see a clear distinction between numbers themselves and the conventional characters, or nine digits, used to represent them. The *number* six is entirely different from the *figure* 6. The child, the savage, or the ignorant man may comprehend numbers, although he may not have the least knowledge of the figures that are used by the arithmetician.

The idea of number begins with the idea of *units*, or single things of which number is composed, and which are necessarily thought of when we ask or answer the question, "how many?"

The unit of a number may consist of a group or groups of other units. Thus, in speaking of a quantity of eggs, we may say, "there are six 30-dozen cases," the unit of the number *six* being *one* 30-dozen case. Each case may then be thought of as 30 units of *one dozen* eggs each. Finally, we may think of each dozen as a number, the unit of which is *one egg*.

After some experience with numbers in their application to the things about us, we are enabled to conceive of the *numbers themselves*, and without any thought of their application to objects. Thus we may think of 6, or 10, or 15, without thinking of any particular unit as the number basis. Numbers thus thought of or expressed are called **Abstract Numbers**.

When we apply numbers directly to definite objects, as in the expressions, *seven dollars*, *five sheep*, *eight dozen eggs*, etc., the numbers are said to be **Concrete**.

NOTE.—Travelers report that the Terra del Fuegians, and certain tribes of the Australian Aborigines, are unable to count beyond five, the number of fingers on one hand. It is a curious historical fact that the fingers (digits) were primarily used by all peoples as a means of indicating number. This is why all number systems use *five* and *ten* as the basis in forming number groups.

There is much difference among individuals in the ability to recognize numbers at sight, and without stopping to count. Some persons cannot recognize more than five objects without counting, while others can recognize eight or nine. It is an interesting experiment to test this power in the case of a class of students, by arranging irregular groups of from five to ten objects, as beans, bits of chalk, kernels of corn, etc., allowing the students to name at sight, as far as they can, the number in each group. The power to see instantly the number of objects in a group is susceptible of cultivation.

Arithmetic, as a *science*, treats of the principles and relations of numbers, and the accepted methods of computing by means of the figures or characters that represent them. As an *art*, it teaches us to apply these principles and methods to the common affairs of life.

The Fundamental Processes of Arithmetic are *Notation*, *Numeration*, *Addition*, *Subtraction*, *Multiplication*, and *Division*.

THE REPRESENTATION AND EXPRESSION OF NUMBERS.

Notation is the method of indicating numbers by means of letters, figures, or other characters.

The representation of numbers by means of special symbols is of very ancient origin—perhaps as old as language itself. Among most ancient peoples numbers were represented by letters of the alphabet. In the Greek and the Hebrew notations, each of the nine digits was denoted by a letter, while other letters represented groups of 10's, as 20, 30, 40, 50, 100, etc. In the Roman system the grouping was carried still further, and fewer characters were used. But far superior to all other systems is the one now in general use, and known as the **Arabic**. This method of number notation probably originated in India in very ancient times. A knowledge of it was acquired by the Arabian mathematicians, and it began to come into use in Europe about the thirteenth century.

The distinctive features of the Arabic system are:

1. The employment of the nine characters or "digits," 1, 2, 3, 4, 5, 6, 7, 8, 9, to denote, respectively, the first nine numbers.
2. The variation of the unit values of these digits according to the position they occupy in the number.
3. The use of the tenth character, 0, to indicate the absence or negation of numerical value.

The unit values of the digits in an Arabic number increase in a tenfold ratio from right to left. Thus, in the number 357, the unit value of the 7 is 1, of the 5 is 10, and of the 3 is 100. Now, if we place a cipher to the right of the number (3570), the unit value of each digit is increased tenfold, since the order of each digit from the right of the number has been increased one place.

By the *order* of a digit is meant the value of its units, as indicated by its position in the number.

The orders for the first sixteen positions are named as follows:

Quadrillions.	Hundred-trillions.	Ten-trilllions.	Trillions.	Hundred-billions.	Ten-billions.	Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.
5	3	6	7	4	7	8	5	2	3	7	1	6	4	3	2

For convenience in reading numbers, the orders are grouped into *periods* of three figures each, and in reading, the unit value of the right hand figure in each period is named, instead of naming the unit value of all the figures. Thus, in reading the above number, 5367478523716432, we do not read, "5 *quadrillions*, 3 *hundred trillions*, 6 *tens trillions*," etc., but we separate the number into periods of three figures each, beginning at the right, thus, 5,367,479,523,716,432, 5 *quadrillion*, 367 *trillion*, 478 *billion*, 523 *million*, 716 *thousand*, 432, the unit value of the unit's period not being expressed.

Numeration is the process of giving expression in words to members that are indicated by figures, letters, or other characters.

To read Arabic numbers readily, one must be able:

(*1*) To read at sight any group of three figures.

(*2*) To name the periods, in their proper order, from highest to lowest.

NOTE.—Strictly speaking there is no limit to the number of periods; but numbers beyond the sixth period, or Quadrillions, are rarely used. The student should learn the periods at least this far, and be able to name them from left to right.

Exercise.

Read at sight the following number groups:

123	101	020	001	905	090	909
328	025	008	075	770	202	060
469	606	206	010	080	003	100
510	011	110	050	709	030	005

In the following numbers the periods are indicated at the head of the columns. Practice reading them rapidly from left to right until you can name the period without looking at the column head. If a period is occupied by ciphers, pass it without mention. Notice that the left-hand period may have less than three figures. Do not name the unit value of the first, or unit's, period.

NOTE.—In reading numbers it is now customary to omit the word "and." Say, "five hundred seventy-six," not, "five hundred *and* seventy-six," "Six hundred two," not "six hundred *and* two."

It is also customary, especially in business offices, to read thousands and hundreds together, as so many hundreds. Thus, instead of saying, "one thousand seven hundred sixty-five," we may say, "seventeen hundred sixty-five, or we may read, 7429, "seventy-four hundred twenty-nine."

Read the following:

QUAD.	TRILL.	BILL.	MILL.	THOUS.	.
160	045	001	206	570	002
14	120	000	567	010	206
506	200	014	000	005	700
870	002	569	703	000	504
9	000	000	010	000	530
	53	010	278	005	500
16	000	000	050	005	020
		500	000	030	003
120	601	250	006	310	037

In the following numbers the periods are indicated by a separating point. Read them.

20,120,160,130,701
150,000,627,100,034
573,020,050,006
5,006,279,130,162
300,000,017,235,837
20,371,500,006,008
40,271,638,798,580,367

Separate the following numbers into periods, beginning at the right, and read them:

127003710025673
800000670001256
1127000008670000124
372607165002016000
530802016300000167

Read the following without pointing off the periods:

1261025
3702010
5000120
31000100
2701630346
2000016307
4020300061
25702016730
400210103
500160710
82000102
3000001200

WRITTEN EXERCISE IN ARABIC NOTATION.

Express in figures the following numbers:

1. Nine hundred, one.
2. One hundred, nine.
3. Two thousand, ten.
4. Eleven hundred, eleven.
5. Sixteen hundred, one.
6. Eighty thousand, ten.
7. Four hundred thousand, eleven.
8. Sixty-two million, twenty-six.
9. Forty-eight hundred, sixty.
10. Three million, forty.
11. Forty million, three.
12. Eleven million, twenty thousand, four.
13. Seven hundred one million, forty thousand, eleven.
14. Ninety million, ten thousand, one.
15. Fourteen billion, seven hundred million, forty thousand, five.
16. Eight billion, eighty.
17. Forty billion, forty million, forty thousand, forty.
18. Seven hundred twenty billion, three thousand, ten.
19. Sixty-five million, eight hundred, four.
20. Ten billion, twenty.
21. One hundred billion, sixteen million, one hundred, ten.

22. One hundred one billion, one hundred ten million, one hundred thousand, ten.

23. Nineteen trillion, fifty billion, forty thousand, four.

24. Seven hundred one trillion, twenty million, fifteen thousand, two hundred fifty.

25. Eight hundred sixty quadrillion, ten trillion, one billion, eleven thousand.

THE ROMAN NOTATION.

This system was in general use throughout Europe until about the thirteenth century, and it is still used to indicate the hours on clock-dials, in numbering the chapters and other divisions of books, and for a few other minor purposes.

In the Roman Notation, all numbers are represented by means of seven letters of the alphabet, and the combinations formed by them.

These seven letters and the numbers represented by them are as follows:

I=One.
V=Five.
X=Ten.
L=Fifty.
C=One Hundred.
D=Five Hundred.
M=One Thousand.

The principles governing the combining of these number-letters are as follows:

1. When a letter is followed by the same letter, or a letter denoting a lower number, the number value of the combination is equal to the sum of the letters. Thus, VI=six; XX=twenty; XVIII=eighteen.

2. When a letter is preceded by a letter denoting a smaller number, the value of the combination is equal to the difference between the numbers. Thus, IV=four; XL=forty; XC=ninety; CD=four hundred; CM=nine hundred.

3. A dash (—) placed over a letter or combination of letters increases their value a thousand-fold. Thus, $\overline{\mathrm{V}}=5000$. $\overline{\mathrm{XV}}\mathrm{CC}=15200$.

Two or more dashes are used to indicate the multiplying of a number by a million, a billion, etc. Thus $\overline{\overline{\text{X}}}$, $\overline{\text{VI}}$, XL=10,006,040.

Since, at the present time, Roman numerals are rarely of never used to indicate large numbers, the study of this notation is a matter of historical, rather than practical, importance. Familiarity with the combinations in the following table will enable the student to read any Roman numeral that he is likely to meet with in modern works.

TABLE OF ROMAN NUMERALS.

I,	1	XV,	15	CC	200
II,	2	XVI,	16	CCC,	300
III,	3	XVII,	17	CD,	400
IV,	4	XVIII,	18	D,	500
V,	5	XIX,	19	DC,	600
VI,	6	XX,	20	DCC,	700
VII,	7	XXX,	30	DCCC,	800
VIII,	8	XL,	40	CM,	900
IX,	9	L	50	M,	1000
X,	10	LX,	60	MM,	2000
XI,	11	LXX,	70	$\overline{\text{X}}$,	10000
XII,	12	LXXX,	80	$\overline{\text{L}}$,	50000
XIII,	13	XC,	90	$\overline{\text{C}}$,	100000
XIV,	14	C,	100	$\overline{\text{M}}$,	1000000

ADDITION.

How many units in the numbers 2, 3, and 4?

Mary spent 3 cents for a pencil, 1 cent for a pen, and 4 cents for a tablet, how many cents did she spend for all? How many letters are required to write the words: "See what God hath wrought?"

The operation of finding a number that is equal to two or more given numbers is called **Addition**.

The numbers to be added are called **Addends**.

The result of the Addition is called the **Sum**.

Addition is indicated by the sign called **Plus** (+), which, when written between two numbers, implies that they are to be added.

The **Sign of Equality** (=) when written between two numbers, or combinations of numbers, implies that they are equal. Thus the expression, "9+6=15," is read, "nine plus six equals fifteen," (or "nine and six are fifteen"), and implies that the two numbers, 9 and 6, are together equal to 15, or, what is the same thing, that the sum of 9 and 6 is 15.

In mathematics, the expression of equality between two or more quantities is called an **Equation**.

PRINCIPLES.

1. Only like numbers, or those having the same kind of units, can be added.

2. The result of an addition is the same, regardless of the order in which the numbers are added.

METHODS AND EXERCISES.

The result of an addition may be found by counting the individual units of the numbers to be added, or by a shorter process, which depends upon a knowledge of the *Addition Table.* This is a table which exhibits the several sums obtained by adding consecutively each of the numbers less than ten to itself, and to every other number less than ten.

In the following form of the Addition Table the several sums are shown at the junctions of the vertical and horizontal columns.

ADDITION TABLE.

1	2	3	4	5	6	7	8	9
2	4	5	6	7	8	9	10	11
3	5	6	7	8	9	10	11	12
4	6	7	8	9	10	11	12	13
5	7	8	9	10	11	12	13	14
6	8	9	10	11	12	13	14	15
7	9	10	11	12	13	14	15	16
8	10	11	12	13	14	15	16	17
9	11	12	13	14	15	16	17	18

Before one can add with any degree of accuracy or facility, he must know the above table perfectly. He should also know all combinations in the Addition Table that are required to produce any sum given in the table. These combinations are shown in the following table, and they should be thoroughly mastered.

TABLE OF COMBINATIONS.

$$\left.\begin{matrix}1+3\\2+2\end{matrix}\right\}=4 \quad \left.\begin{matrix}1+4\\2+3\end{matrix}\right\}=5 \quad \left.\begin{matrix}1+5\\2+4\\3+3\end{matrix}\right\}=6 \quad \left.\begin{matrix}1+6\\2+5\\3+4\end{matrix}\right\}=7 \quad \left.\begin{matrix}1+7\\2+6\\3+5\\4+4\end{matrix}\right\}=8$$

$$\left.\begin{matrix}1+8\\2+7\\3+6\\4+5\end{matrix}\right\}=9 \quad \left.\begin{matrix}1+9\\2+8\\3+7\\4+6\\5+5\end{matrix}\right\}=10 \quad \left.\begin{matrix}2+9\\3+8\\4+7\\5+6\end{matrix}\right\}=11 \quad \left.\begin{matrix}3+9\\4+8\\5+7\\6+6\end{matrix}\right\}=12 \quad \left.\begin{matrix}4+9\\5+8\\6+7\end{matrix}\right\}=13$$

$$\left.\begin{matrix}5+9\\6+8\\7+7\end{matrix}\right\}=14 \quad \left.\begin{matrix}6+9\\7+7\end{matrix}\right\}=15 \quad \left.\begin{matrix}7+9\\8+8\end{matrix}\right\}=16 \quad \left.\begin{matrix}8+9\end{matrix}\right\}=17 \quad \left.\begin{matrix}9+9\end{matrix}\right\}=18$$

Numbers to be added are usually written in vertical columns, units of the same order being written in the same column.

MENTAL EXERCISE.

Read at sight, and as rapidly as possible, the results of the following additions. Read first from right to left, then from the top to the bottom, and then from the bottom to the top.

2	8	5	6	9	3	4	7	9	7
9	6	3	4	6	7	2	8	9	2
6	5	6	5	3	9	4	9	7	5
2	9	6	2	6	7	6	5	4	5
3	5	6	9	4	9	3	8	2	6
9	4	8	3	5	2	5	9	6	9
6	9	9	8	4	7	9	6	7	5
7	1	8	5	7	5	9	4	9	8
9	8	2	7	5	5	8	2	4	6
4	7	8	6	8	6	6	7	9	9

CONSTANT UNIT FIGURE.

The unit figure obtained by adding numbers having the same unit figure is constant. Thus, since the unit figure of the sum of 6 and 9 (15) is 5, the unit figure obtained by adding any two numbers ending respectively in 6 and 9 will be 5; as, 19+6, 26+9, 49+6, 39+16, etc. In like manner, the unit figure for 4 and 7 is 1; for 8 and 5 is 3; for 9 and 3 is 2; and so on.

MENTAL EXERCISE.

Name as rapidly as you can the unit figures produced by adding the following pairs of numbers. Do not name the result; merely give its unit figure.

6 and 9
4 and 6
7 and 8
6 and 7
5 and 6
3 and 8
9 and 9
7 and 4
2 and 8
9 and 5

19 and 7
26 and 9
47 and 4
16 and 8
27 and 5
84 and 6
28 and 9
58 and 5
36 and 7
49 and 2

54 and 27
39 and 26
45 and 29
98 and 17
26 and 55
44 and 58
17 and 46
53 and 68
25 and 37
28 and 52

Name rapidly the results of the following:

16	8	27	19	35	29	9	28	8	59
7	14	5	6	8	7	34	7	15	6

14	33	6	9	38	4	67	55	27	58
8	7	56	42	3	76	9	7	4	6

26	15	47	8	9	19	28	3	44	29
7	9	6	22	64	7	5	79	8	9

CONTINUOUS ADDITIONS.

By a *continuous addition* is meant the successive addition of the same number, as 6+7+7+7+7 or 2+5+5+5+5, etc. Rapid continuous additions are a most helpful means of aiding one to add rapidly and accurately.

Note. In adding orally a series of numbers, simply announce results; do not say, "5 and 7 are 12 and 5 are 17 and 5 are 22," etc.; say, "5, 12, 17, 22."

MENTAL EXERCISE.

1. Begin with 6 and add 5's until you get a result of 56.

2. Commencing with 9, add 7's until you reach a number whose unit figure is 9.

3. Commencing with 4, add 6's until you have a sum greater than 100.

4. Commence with 2 and add 9's continuously until the sum exceeds 100.

5. Beginning with 5, add 7's until a number is obtained whose unit figure is 5.

In the following exercises continue the additions until the results exceed 100:

1. Commence with 3 and add 8's.
2. " " 9 " " 6's.
3. " " 1 " " 7's.
4. " " 3 " " 4's.
5. " " 7 " " 9's.
6. " " 6 " " 8's.
7. " " 3 " " 7's.
8. " " 2 " " 3's.
9. " " 5 " " 8's.
10. " " 4 " " 7's.

ADDING SINGLE COLUMNS.

In the following exercise, add the numbers one at a time, commencing at the bottom and announcing the result as each number is added. After recording the final result, verify by adding the column again, beginning at the top. Add rapidly and with certainty, and allow nothing to divert the attention until the result is obtained.

WRITTEN PRACTICE.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
4	7	3	7	9	8	4	3	4	3	9
3	4	6	2	2	4	3	9	7	6	5
8	5	2	9	3	2	9	8	6	0	8
6	6	8	8	6	9	7	4	3	7	6
3	3	4	6	8	8	8	6	5	2	7
5	2	9	3	4	7	9	2	8	5	4
4	4	6	8	9	9	7	9	2	6	9
2	9	5	4	7	8	6	8	7	9	3

(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
8	7	9	7	3	2	5	9	6
5	6	7	3	9	9	9	2	9
6	5	5	8	4	3	3	7	5
7	3	3	6	8	8	6	3	7
4	4	2	5	6	4	7	5	2
9	6	4	9	5	7	2	8	8
3	2	6	2	2	6	4	4	4
6	9	8	4	4	5	9	1	9
2	8	3	6	7	8	1	6	3
4	3	5	3	9	3	6	9	7
6	6	9	8	3	9	7	7	6
8	7	7	7	6	2	3	3	2
9	5	6	6	4	7	8	6	8
5	4	2	2	8	3	6	7	1
8	8	6	5	9	5	9	9	5
3	3	5	9	1	9	2	4	6
0	7	8	3	7	6	7	1	9
7	2	9	7	6	8	8	6	8
6	6	3	4	8	4	6	8	6
9	4	8	9	4	7	3	5	9

ADDING NUMBERS OF TWO OR MORE ORDERS.

Carrying the Tens.—In adding numbers composed of two or more orders, the unit's column is added first, and if the sum of this column exceeds 9, the unit figure in recorded and the tens are "carried," or added with the numbers of the next column. In like manner, if the sum of the ten's column exceeds 9, the hundreds are carried to the next column, and so on with the remaining columns.

WRITTEN PRACTICE.

Copy neatly the following numbers, add them, and prove the results by adding from top to bottom:

(21)	(22)	(23)	(24)	(25)	(26)
364	327	643	842	634	3426
742	645	864	375	784	3548
436	327	963	842	643	7263
743	846	327	963	426	4986
375	364	842	643	962	5432
					8374
					3652

(27)	(28)	(29)
4	4983649	9
26	837463	84
342	54289	356
5463	7436	8974
79834	842	765
378426	97	98
5897634	8	6

(30)	(31)	(32)	(33)	(34)
21524	34253	22736	13125	12567
13065	25321	16853	24143	45526
43753	12579	40632	13560	30415
10476	40055	12345	30234	57939
44342	54204	43210	45746	21473
13526	13652	25607	25612	65114
26437	31148	34051	14200	13496
10596	26554	15779	17488	28744

RECORDING PARTIAL ADDITIONS.

In adding numbers composed of several columns of figures, accountants are liable to be interrupted before the result is obtained, and thereby have to do their work all over. To obviate this, the sum of the several columns may be recorded separately, as illustrated in the accompanying example.

46976	45
58764	49
74256	56
9845	54
47876	27
895	
6438	
29645	
274695	

EXPLANATION.—The first number, 45, is the footing of the first column, giving us 5 to write under units' column, and 4 to carry to tens' column. 49 is the total of the second column, including the 4 tens carried. After the footings of all the columns are found and proven, the result is read, beginning with the last number and reading up the units' column (274695). This result is then written as the required footing.

Add the following, recording separate results, as illustrated in the foregoing example:

(35)	*(36)*
56432156	$1242.46
75439965	1154.22
28365283	2425.12
96543216	3204.25
76896543	1115.38
76987654	2250.50
76345923	1425.42
91683468	1350.24
23456837	1556.18
	2345.15
	2412.20
	3525.32

GROUPING.

The learner may greatly increase the rapidity of his additions through the process known as *grouping*. This consists in learning to recognize at a glance the sums of various groups of figures, and to add them as one number. The ability to do this depends upon practice and the possession of a "quick eye." Begin by learning to recognize all the groups of two figures whose sum is 10. These are as follows:

1	2	3	4	5
9	8	7	6	5

The "nine groups" are also easy to add, since adding nine to any number diminishes the unit figure by one.

The nine groups are $\left\{\begin{matrix}1\\8\end{matrix}\right.$ $\left\{\begin{matrix}2\\7\end{matrix}\right.$ $\left\{\begin{matrix}3\\6\end{matrix}\right.$ $\left\{\begin{matrix}4\\5\end{matrix}\right.$

In the first two of the following examples the number-groups are inclosed in braces; treat each of these groups as a single number. In the remaining examples form your own groups.

(37)	(38)	(39)	(40)	(41)	(42)
3	8	2	9	7	$257.32
6	5	7	3	8	242.28
7	4	8	6	2	420.50
8	7	3	5	6	134.36
2	9	4	5	3	175.25
5	1	6	8	4	200.00
3	6	2	7	1	324.14
7	3	7	2	8	455.50
2	2	3	6	7	255.75
4	7	5	3	6	125.80
3	7	4	4	0	137.25
7	0	8	1	4	143.56
6	6	6	0	3	
3	4	9	7	9	
0	8	3	3	1	
7	7	7	8	5	
9	2	2	1	4	
2	6	5	2	8	
6	3	4	5	2	
4	7	7	4	7	
				8	

(43)	(44)
$524.25	$1242.36
247.19	1154.22
325.30	2425.12
432.75	3204 25
157.45	1115.38
284.50	2250.50
356.15	1425.42
575.25	1350.24
142.75	1556.18
381.54	2345.15
434.34	2412.20
150.50	3525.32

ADDING HORIZONTAL COLUMNS.

It is frequently desirable to add numbers that are written in horizontal columns, and without copying them.

Find the results of the following additions without copying the numbers:

WRITTEN PRACTICE.

45. 8+7+6+14+27+46=?

46. 42+6+19+21+35+26=?

47. \$1.15+\$5.00+\$3.75+\$2.50+\$1.25=?

48. 65¢+\$2.45+75¢+\$10.00+\$7.30+85¢=?

49. 765+528+326+467=?

50. \$2.45+40¢+15¢+\$2.00+\$4.45+\$5.00+60¢=?

51. \$21.50+\$19.00+75¢+\$35.25+\$4.00+\$13.00+5.00=?

52. \$9.40+\$7.15+\$21.25+\$8.00+65¢+\$3.20+\$5.00=?

53. \$8.05+\$41.50+\$11.00+25¢+70¢+\$9.65+\$14.00+\$1.35=?

54. \$20.00+\$1.40+\$28.50+90¢+\$9.25+\$1.15+\$82.00=?

WRITTEN PROBLEMS.

55. The following are the items of cost for Mr. Granger's new residence property: Cost of lot, \$1875; paid for lumber, \$626.06; carpenter work, \$314.25; masonry, \$175.20; plastering, \$85.00; plumbing and wiring, \$93.05; windows and doors, \$45.10; painting, \$76.00; paper hanging, \$41.75. What was the total cost of the property?

56. Frank's yearly allowance for college expenses is as follows: Tuition and fees, \$150; board, \$175; clothes, \$75; books, \$25; laundry, \$30; stationery, \$5; car fare, \$35; lectures and amusements, \$25; incidentals, \$15. What was his total yearly allowance?

57. A dealer delivered ten loads of coal with recorded weights as follows: 2142 lbs., 1956 lbs., 2047 lbs., 2265 lbs., 2143 lbs., 1989 lbs., 2008 lbs., 2106 lbs., 1989 lbs., 2047 lbs. What was the total weight?

58. The seven townships of Jefferson county show school enrollments as follows: Merrimac, 567; Clinton, 906; Walnut, 1146; Coldwater, 428; Marion, 1019; Rock, 479; Harding, 1428. Find the total enrollment for the county.

59. Find the sum of all the odd numbers from 0 to 50.

60. Find the sum of all the even numbers from 0 to 51.

61. Find the sum of:

One hundred twenty-five.
Three hundred thousand, two hundred sixty-four.
One thousand, seven.
Fifty thousand, five hundred twenty-six.
Twenty-seven.
Three hundred ninety-six.
Seven hundred seventy-five.
Three thousand, three hundred thirty-three.

62. Add:

7 hundred 25.
3 thousand, 7 hundred 45.
8 thousand, 9 hundred 64.
5 thousand, 9 hundred 84.
9 thousand, 3 hundred 26.
8 thousand, 17.
4 thousand, 37.
8 hundred 24.
3 thousand, 28.

63. The following table represents the total daily attendance of the public schools of Hamilton Township for the ten school months of 1899–1900. Copy carefully this tabulation on a sheet of blank paper, add the columns and fill in the total for each month and for each school. The total attendance for the year is entered in the last space to the right, and it should be proved by adding the totals for the different months, also for the several schools.

SCHOOL	Sept	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June	TOTALS
Lincoln	1126	1019	987	928	1097	1065	989	1074	994	917	
Washington	1049	984	946	898	1036	978	943	960	917	889	
Longfellow	876	807	758	716	847	827	810	864	793	719	
Douglass	948	892	868	807	849	808	796	757	714	658	
Clay	1075	1043	985	924	1019	981	967	1007	947	939	
Fulton	829	757	749	717	837	806	795	801	824	783	
Franklin	743	704	685	612	728	694	709	716	697	651	
Whittier	843	807	784	757	838	816	824	775	786	727	
Totals											

SUBTRACTION.

What is the sum of 4 and 3? If, from a pile of 7 pennies, 3 pennies are taken away, how many will remain? If from the sum of 8 and 6, 6 be taken away, what will remain? If 8 be taken from the sum of 9 and 8, what number will remain? If from the sum of any two given numbers we take one of them away, what remains? If from a basket containing 12 apples we remove 5 apples, what remains?

DEFINITIONS.

Subtraction is the process of finding the excess of one number over another of the same kind, or of finding the difference between two numbers.

The **Minuend** is the greater of the two numbers.

The **Subtrahend** is the less number.

The **Difference**, or **Remainder**, is the result of the subtraction, or the excess of the Minuend over the Subtrahend.

The **Sign of Subtraction** is the dash (–); it is read "minus," and when written between two numbers implies that the number to the right of the sign is to be subtracted from the number to the left. Thus, $12-4=8$.

PRINCIPLES.

1. The Subtrahend and Minuend must be like numbers.

2. The Minuend is equal to the sum of the Subtrahend and Remainder.

3. Increasing or diminishing both Minuend and Subtrahend by the same number does not affect the value of the Remainder.

If from the sum of two numbers one of them be subtracted, the result is the remaining number. It is therefore evident, (1) that subtraction is the reverse process of addition, and, (2) that to subtract readily one must be thoroughly familiar with the addition table.

Thus, if one knows that $7+8=15$, he will also know that $15-7=8$, and that $15-8=7$.

MENTAL EXERCISE.

Read the results, as rapidly as you can, of the following indicated subtractions:

12	16	18	17	9	8	11	13	15	18	11	9	7	12
6	9	9	8	6	5	4	4	7	9	6	4	3	7

17	13	16	16	14	9	13	14	18	17	10	12	10	14
9	5	7	8	6	5	8	9	9	8	8	4	6	7

12	13	16	14	13	11	13	10	17	12	14	13	15	11
8	9	8	8	6	7	7	3	9	5	5	4	6	5

Without copying the numbers, write as rapidly as you can the results of the following subtractions:

(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*
67	96	26	43	85	28	64	79	87	58
24	32	14	21	52	17	43	54	35	26

(11)	*(12)*	*(13)*	*(14)*	*(15)*	*(16)*	*(17)*	*(18)*	*(19)*	*(20)*
84	96	37	48	56	27	75	59	37	65
41	52	25	13	25	13	52	24	16	23

GENERAL EXPLANATION OF THE SUBTRACTION PROCESS.

First Example
6579
2343
4236

Where the digits of the minuend are all greater than the corresponding digits of the subtrahend, the subtraction process is quite simple, and the result can be ascertained by a mere inspection of the numbers. In this case, we may begin the subtraction either at the left or right.

Second Example
7643
2717
4926

In this example some of the digits of the subtrahend are greater than the corresponding digits of the minuend. In such cases the subtraction necessarily is commenced at the unit order. The number of units indicated in the unit's column of the minuend, 3, is less than the unit's number in the subtrahend, 7; we therefore add 10 to the minuend figure, making 13 units, from which we subtract the 7, writing the result, 6, in the unit's place of the remainder. We may now assume that the 10 which we added to the unit figure of the minuend was "borrowed," or taken away from the tens' column, and we may consider that the 4 tens have been reduced to 3, from which we now take the 1 ten in the subtrahend, writing the remainder, 2, in the tens' column of the remainder. Or we may apply Prin. 3, and add, or "carry," as it is sometimes called, 1 ten to the tens of the subtrahend, making 2 tens, which we subtract from the 4 tens of the minuend, leaving 2 tens to write in the remainder as before. Both of these methods are employed, but the result is the same in either case.

Proceeding in the same way in the case of the hundreds, we add 10 hundreds (1 thousand) to the 6 in the minuend, making 16, from which we subtract the 7 of the subtrahend, writing the result 9 in the remainder.

Then, either considering the 7 thousand of the minuend diminished to 6, or the 2 thousand of the subtrahend increased to 3, we subtract, writing the result, 4, in the remainder

PROOF OF SUBTRACTION.

Since the remainder is the difference between the subtrahend and the minuend, the minuend is equal to the sum of the subtrahend and remainder; hence,

TO PROVE SUBTRACTION.—*Add the Subtrahend to the Remainder; if the sum is equal to the Minuend, the subtraction is correct.*

WRITTEN EXERCISE.

Solve the following and prove your results.

1. 9124 – 857 = ?
2. 2147 – 1429 = ?
3. 24582 – 9385 = ?
4. 82430 – 29572 = ?
5. 940367 – 47038 = ?
6. 102012 – 87965 = ?
7. 720164 – 385726 = ?
8. 80000127 – 5000864 = ?
9. \$17965.42 – \$12867.92 = ?
10. \$100046.21 – \$74506.38 = ?
11. \$1006.20 – \$95.87 = ?
12. 2964301 – 780067 = ?
13. 42716301 – 743986 = ?

Find the difference between—

14. 4275 and 958.
15. 1256 and 8794.
16. \$2796.42 and \$37601.15.
17. \$1215627 and \$862479.
18. 3874216 and 982475.
19. 10201670 and 34876216.
20. 937624 and 1000000.

COMPLEMENTARY NUMBERS.

Complementary Numbers are any two numbers whose sum is equal to a unit of the next higher order. Thus, 6 and 4, 7 and 3, 8 and 2, etc., are complementary numbers, as the sum of each of these pairs is 10; and 36 and 64, 27 and 73, 42 and 58, are complementary, as the sum of each pair is 100.

A number is said to be the *complement* of another when the sum of the two is a unit of the next higher order; thus, 7 is the complement of 3, 26 is the complement of 74, and 364 is the complement of 636.

In all complementary numbers of more than one figure, the sum of the unit figures is 10, while that of the other corresponding orders is 9

Example:	4632	247
	5368	753
	$999_{10} = 10000$	$99_{10} = 1000$

By applying the foregoing principle, a little practice will enable one to name, at sight, the complement of any number. The ability to do this is very useful in the business office in making change, etc.

MENTAL EXERCISE.

Name as rapidly as you can the complements of the following numbers: 36, 26¢, 64, 73, 28¢. 83, 42, 34, 65¢, 48, $26, 36, 57, $21, 32, 63, 76, 19, 83, 55, 22, 31, 43, 82, 53.

Give the complements of the following: 125, 236, 328, 475, 643, 764, 238, 753, 146, 321, 816, 458, 735, 593, 637.

In case a ten-dollar bill is offered in payment, name the amount of change required for each of the following amounts: $2.75, $1.45, $3.56, $8.27, $3.42, $6.29, $4.53, $2.35, $5.75, $8.21, $6.15, $4.38, $7.23, $2.91, $3.54, $2.74, $8.75, $4.18, $3.37, $2.85.

MENTAL EXERCISE.

In each of the following problems, find the change required by subtracting the sum of the items from the sum paid. Where possible, find the result mentally.

	Items.	Paid.		Items.	Paid.
1.	15¢, 12¢, 50¢,	$1	*8.*	75¢, 24¢, 47¢,	$2
2.	24¢, 36¢, 18¢,	$1	*9.*	$1.25, 60¢, 22¢,	$5
3.	45¢, 15¢, 36¢,	$2	*10.*	$3.25, 50¢, $1.25,	$5
4.	21¢, 18¢, 12¢,	$1	*11.*	$4.60, $2.25, $1.20	$10
5.	45¢, 55¢, 48¢,	$2	*12.*	$1.20, $1.50, $2,	$5
6.	23¢. 34¢, 25¢,	$1	*13.*	$3.60, $4, 55¢,	$10
7.	63¢, 8¢, 4¢,	$1	*14.*	$4.25, $5.50, $8.20,	$20

WRITTEN PROBLEMS.

21. What number must be added to 98647 to give a result of 102613?

22. From a carload of coal containing 46975 lbs., 29874 lbs. has been sold; how much coal remains in the car?

23. On Jan. 1, Carter & Stone's bank balance amounted to $28596.15. On Feb. 1 their balance was $17524.98; what was the amount of decrease during January?

24. The consumption of water by the town of Clyde for June, 1900, amounted to 13,247,162 gallons. The consumption for July was 15,964,386; what was the increase in consumption over the previous month?

25. The purchase price of the Brockton Street Car Plant was $275,946.25, of which sum, $198,596.48 has been paid. What sum remains unpaid?

26. In 1890 the unsold government land in one of the Western States was 526,516 acres. In 1900 the unsold land was 359,879 acres. What number of acres was sold during the decade?

27. What number must be subtracted from 98601 to give a remainder of 29748?

WRITTEN PROMISCUOUS PROBLEMS.

28. On Nov. 1, Fay & Brookings' balance at the Mercantile Bank was $947.60. On Nov. 10 they deposited $256.75, and on Nov. 27, $425.83. During the month they withdrew on their checks, $1159.25. What is their bank balance on Dec. 1?

29. The school population of Jackson County for 1900 was 21568. Of this number of children, 2594 are in Holton, 2862 are in Bridgeport, and 1846 in Vinton, the three leading towns in the county. What is the school population of the remainder of the county?

30. The total school appropriation of the town of Lawndale for 1900 is $12,538.67. During the year, school warrants were issued as follows: teachers' salaries, $8528.50; salaries of other officers, $2847.50; repairs, $469.15; fuel and incidentals, $1087.25; library, $287.60. What is the amount of the deficiency for the year?

31. The outlays of the Oakdale Manufacturing Co. for the year ending July 1, 1900, were as follows: wages and salaries, $21,564.42; materials and equipment, $19462.75; advertising, $3586.15; other expenses, $642.90. If the gross receipts were $58962.70, what were the net receipts for the year?

ACCOUNTS.

In bookkeeping the various charges and credits relating to one person or firm, or to one class of transactions, are assembled in groups called *accounts.*

The accompanying form represents an account with Hall & Morton. That part of the account at the left of the central dividing line is called the *debit side* of the account, and that part to the right is called the *credit side.* The various sums or items on the left are called *debits*, and those on the right are called *credits.*

The difference between the sums of the debits and credits is called the *balance* of the account.

To *close* or *balance* an account, is to enter the balance on the lesser side and rule up the account, as shown in the accompanying form.

When an account is properly closed, the totals or footings of the two sides should be equal.

HALL & MORTON.

1900				(DEBITS)		1900				(CREDITS)	
Jan...	2	Mdse.....	...	126	50	Jan..	6	Cash......	....	50	00
" ...	8	"	...	254	25	" ..	12	Mdse.....	....	162	25
" ...	21	Cash......	...	75	00	" ..	29	"	....	64	90
Feb..	4	Mdse.....	...	115	75	Mar.	1	Balance...	....	512	90
" ...	16	"	...	45	10	......	..	....	....	......	...
" ...	21	Cash......	...	50	00	......	..	....	....	......	...
" ...	23	Mdse.....	...	123	45	......	..	....	....	......	...
				790	05					790	05

WRITTEN PRACTICE.

In the following examples only the money columns of the accounts are shown. Prepare similarly ruled forms for these accounts, copy them neatly, after which ascertain the balances, and close and rule up the several accounts:

(32)

M. S. Robinson.

349	60	592	11
74	68	821	65
148	24	273	87
205	87	21	59
		143	25
		65	27
		9	46

(33)

C. W. Marsh.

142	50	253	40
275	25	19	28
8	15	446	27
34	68	35	92
	45		
519	12		

(34)

Cash.

79	54	62	95
427		321	60
4	49	11	45
37	58	10	
96		50	
125		43	69
9	14	27	53
329	50	148	65
49	23		
75	80		

(35)

Chester I. Long.

48	75	79	50
15	25	25	
98		367	98
114	60	14	46
85	07	5	
9	25		94
2	46		
300			
48	76		

(36)

C. B. Renfro & Co.

857	28	185	90
9	46	72	47
350		9	80
1754	87	1246	70
957	20	189	45
89	56	968	
		40	08
		3475	62
		8	97
		400	
		98	58

(37)

Bills Receivable.

624	80	250	
3658	70	742	80
459	25	624	80
97	15	75	
500		146	
1076	28	97	15
746	25	2500	
19	15		
5			
253	96		
874	28		

Prepare account-forms for the following schedule of items, and close and rule up the several accounts. In entering the items omit dollar marks.

(*38*) C. J. RAMSDELL.

DEBITS. $1940.00, 75¢, $142.60, $189.15, $47.25, $54.96, 98¢, $78.00.

CREDITS. $44.60, $15.00, $175.00, $34.75, $5.00, $21.40.

(*39*) CASH.

DEBITS. $39.65, $147.19, $278.47, $435.86, $79.28, $164.00, $358.60, $29.56, $85.90.

CREDITS. $57.24, $94.15, $68.75, $29.00, $58.64, 47¢, $2.78, $13.09.

(*40*) BILLS PAYABLE.

DEBITS. $75.00, $54.28, $47.90, $175.00, $243.80, $67.00, $48.50, $374.19, $59.60, $178.50.

CREDITS. $178.50, $75.00, $59.60, $175.00, $243.80, $54.28, $47.90, $67.00, $374.19, $427.95, $67.75.

MULTIPLICATION.

At 3 cents each, what is the cost of 7 pears?

SOLUTION.—If the pears were bought one at a time, we should have to pay out 3 cents *seven times*, and the cost would equal $3+3+3+3+3+3+3=21$ cents. If, when we add the seven 3's, we remember the result, the next time we have a similar problem we would not need to add, but would simply say, "The pears will cost 7 times 3 cents, or 21 cents."

The **Multiplication Table** is a table showing the results of adding equal numbers. The process of finding the sum of several equal numbers by means of this table is called **Multiplication.**

DEFINITIONS.

In multiplication, the several numbers to be added are thought of and spoken of as one number, which is called the **Multiplicand.** The number that indicates the number of times the multiplicand is taken, is called the **Multiplier**.

The result of the multiplication is called the **Product.**

Thus in the expression, "5 times 9 are 45," 9 is the *multiplicand*, 5 is the *multiplier*, and 45 is the *product.*

The multiplier and multiplicand are also called "factors."

The sign of multiplication is (×), and when written between two numbers indicates that either one is to be taken as an addend as many times as there are units in the other. Thus, the expression "$4\times6=24$" may be read "four times six are twenty-four," or "four multiplied by six equals 24."

Continued Multiplication is the process indicated by joining a series of numbers with the multiplication sign, as, $5\times2\times3\times4$. The result of a continued multiplication is obtained by multiplying any of the numbers by any other, and this product by the third, and so on; the result being the same, regardless of the order in which the numbers are taken.

The number of distinct multiplications indicated in any continued multiplication is equal to one less than the number of factors. Thus, to find the product of 5×7×8×9, three distinct multiplications are necessary.

PRINCIPLES.

1. The multiplier is always an abstract number.

2. The multiplicand and product are like numbers.

3. Either term of the multiplication is contained in the product as many times as there are units in the other term.

MENTAL EXERCISE.

1. Prove by addition that 8 times 7 is equal to 7 times 8.

2. What is the product of 2×3×5×7?

In how many ways can you get the result?

3. Find, by addition, the cost of 5 acres of land at $25 per acre.

4. Name the multiplier, the multiplicand, and the product in the following problem: What is the total length of seven boards, if each board is 9 ft. long?

5. Find, by subtraction, the number that must be multiplied by 7, to give 91 as a product.

6. Find, by addition, the number that 14 must be multiplied by to give 84 as a product.

7. Find, by addition, the product of 3×4×5.

8. How many distinct multiplications are necessary to find the product of 6×7×8×4×12?

9. Prove that the product of 6×3×5×2 is the same regardless of the order in which the several multiplications are performed

10. If $2\times4\times(x)=72$, find x by subtraction.

THE MULTIPLICATION TABLE.

The Multiplication Table, as ordinarily presented, includes the numbers from 1 to 12, the combinations under each of the several numbers being placed in separate groups.

The Multiplication Table here presented contains all combinations from "1×1" to "25×25." To use this table, find the junction of the required columns; thus, if we wish to know the product of 12 times 19, we find it to be 228, which is at the junction of the 12th and 19th columns.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**

SUPPLEMENTARY MULTIPLICATION TABLE.

As a means of enlarging the student's knowledge of the composition of the lesser numbers, we here present an extension of the Multiplication Table, which includes all possible multiplication combinations, the results of which do not exceed 100. Thus the "2 line" is extended to 2×50; the "3 line," to 3×33; the "4 line," to 4×25, and so on.

A knowledge of this supplementary table will prove of the greatest advantage in the student's subsequent arithmetical work, and its thorough mastery is strongly advised.

Note.—The table, as here presented, is shortened by omitting the reverse combinations, the "4 line" beginning with "4x4," the "5 "line," with "5x5," etc. The student should be required to give the results of the several combinations, entirely regardless of the order in which the factors are presented.

SUPPLEMENTARY MULTIPLICATION TABLE.

2× 2= 4
2× 3= 6
2× 4= 8
2× 5=10
2× 6=12
2× 7=14
2× 8=16
2× 9=18
2×10=20
2×11=22
2×12=24
2×13=26
2×14=28
2×15=30
2×16=32
2×17=34
2×18=36
2×19=38
2×20=40
2×21=42
2×22=44
2×23=46
2×24=48
2×25=50
2×26=52
2×27=54
2×28=56
2×29=58
2×30=60
2×31=62
2×32=64
2×33=66
2×34=68
2×35=70
2×36=72
2×37=74
2×38=76
2×39=78
2×40=80
2×41=82
2×42=84
2×43=86
2×44=88
2×45=90
2×46=92
2×47=94
2×48=96
2×49=98
2×50=100

3× 3= 9
3× 4=12
3× 5=15
3× 6=18
3× 7=21
3× 8=24
3× 9=27
3×10=30
3×11=33
3×12=36
3×13=39
3×14=42
3×15=45
3×16=48
3×17=51
3×18=54
3×19=57
3×20=60
3×21=63
3×22=66
3×23=69
3×24=72
3×25=75
3×26=78
3×27=81
3×28=84
3×29=87
3×30=90
3×31=93
3×32=96
3×33=99

4× 4=16
4× 5=20
4× 6=24
4× 7=28
4× 8=32
4× 9=36
4×10=40
4×11=44
4×12=48
4×13=52
4×14=56
4×15=60
4×16=64
4×17=68
4×18=72
4×19=76
4×20=80
4×21=84
4×22=88
4×23=92
4×24=96
4×25=100

5× 5=25
5× 6=30
5× 7=35
5× 8=40
5× 9=45
5×10=50
5×11=55
5×12=60
5×13=65
5×14=70
5×15=75
5×16=80
5×17=85
5×18=90
5×19=95
5×20=100

6× 6=36
6× 7=42
6× 8=48
6× 9=54
6×10=60
6×11=66
6×12=72
6×13=78
6×14=84
6×15=90
6×16=96

7× 7=49
7× 8=56
7× 9=63
7×10=70
7×11=77
7×12=84
7×13=91
7×14=98

8× 8=64
8× 9=72
8×10=80
8×11=88
8×12=96

9× 9=81
9×10=90
9×11=99

10×10=100

MENTAL PROBLEMS FOR ANALYSIS.

1. What will 17 lemons cost at 5 cents each?

ANALYSIS.—They will cost 17 times 5 cents, or 85 cents.

NOTE.—In giving the analysis of problems in multiplication, do not reverse the terms in the expression. Take for the multiplicand the number that is of the same denomination as the required product. Thus, in the foregoing problem it would be illogical to say "five times seventeen," instead of "seventeen times five," notwithstanding that both expressions involve the same numerical result.

2. Allowing 8 rails to the panel, how many rails will be required for 12 panels of fence?

3. If 15 quires of paper are used in one copy of a certain book, how many quires will be used in an edition of 3000 copies?

4. How many days are there in 13 weeks?

5 Find the value of 5 acres of land at $17 an acre.

6. How many eggs in 2 cases containing 30 dozen each?

7. If there are 16 ounces in a pound, find the weight, in ounces, of 6 pounds of nails.

8. What is the value of 13 cases of berries at $3 each?

9. Find the cost of 5000 ft. of lumber at $19 per thousand.

10. What is the value of 11 yards of ribbon at 12 cents a yard?

11. How far will a man travel in 9 hours at the rate of 60 miles an hour?

12. At 25 cents an hour, what can a man earn in 3 days of 8 hours each?

13. What is the distance around a square field that measures 17 rods on each side?

14. If a boy runs 12 feet per second, how far can he run in a minute?

15. In an orchard there are 12 rows of trees with 9 trees in each row; how many trees in the orchard?

16. A room is 60 ft. long and 40 ft. wide; how many square feet of floor space in the room?

NOTE.—The square feet in any surface is equal to the product of the length by the width. The analysis is as follows:

"If the surface were 60 feet long and 1 foot wide, the surface would equal 60 square feet; therefore, a surface 60 feet long and 40 feet wide will contain 40 times 60 square feet, or 2400 square feet.

17. Find the cost of 8 quarts of berries at 5 cents a pint.

18. What is the length of 5 panels of fence, the length of a panel being 16 ft.?

19. Mary's flower bed is 14 ft. long and 6 feet wide, how many square feet in its surface?

20. What is the value of 5 dozen eggs, each egg being worth 2 cents?

21. A certain room is 16 feet long and 10 feet wide, what is the entire distance around the room? How many square feet in the floor?

22. Find the cost of 15 tons of coal at $5 per ton.

A **Multiple** of a number is any number that is obtained by multiplying it by another whole number. Thus, 24, 36, 48, and 60 are each multiples of 12; and 16, 24, 32 and 40 are multiples of 8.

MENTAL EXERCISE.

1. Name, in order, all of the multiples of 12 less than 100. Of 13; of 18; of 16.

2. Of what numbers is 60 a multiple? 72? 96? 85? 68?

3. Name a number that is a multiple of 5 and 12. Of 9 and 15; of 18 and 4; of 7 and 13.

4. Name all multiples, less than 100, of the following numbers: 7, 14, 15, 17, 19, 21, 29.

WRITTEN EXERCISE.

Multiply 5679 by 6.

In complete form, the operation would be recorded as follows:

$$\begin{array}{r} 5679 \\ 6 \\ \hline 54 \\ 42 \\ 36 \\ 30 \\ \hline 34074 \end{array}$$

each figure of the multiplicand being multiplied by the multiplier, the several results being written in proper order and added.

In practice, however this process is shortened by performing the additions mentally as the multiplication proceeds, only the general product being recorded, thus:

$$\begin{array}{r} 5679 \\ 6 \\ \hline 34074 \end{array}$$

In this process, the unit figure of the first multiplication, 4, is written, and the tens are "carried," or added in with the next ten's product, and so on, until all of the digits have been multiplied.

1. Multiply each of the following numbers by 4: 964; 7446; 9241; 24563; 9768.

2. Multiply the following by 5: 2046; 7487; 92507; 70906; 19867.

3. Multiply the following by 6: 7438; 20906; 38720; 84769; 372067.

4. Multiply the following by 7: 42876; 89735; 48738; 90764; 58678.

5. Multiply the following by 8: 9675; 6958; $97.65; $189.48; $9476.28.

6. Multiply the following by 9: 87¢; $9.85; $597.62; $9839.46; 46758.

MENTAL EXERCISE.

Perform mentally as many as you can of the following multiplications. If you have any difficulty, study the Supplementary Multiplication Table.

1. 13×3=?
2. 15×6=?
3. 19×2=?
4. 16×6=?
5. 14×5=?
6. 13×6=?
7. 29×2=?
8. 24×3=?
9. 17×4=?
10. 46×2=?
11. 18×4=?
12. 23×3=?
13. 19×5=?
14. 15×5=?
15. 36×2=?
16. 28×3=?
17. 14×6=?
18. 18×5=?
19. 13×7=?
20. 23×4=?
21. 350×2=?
22. 1900×4=?
23. 18×30=?
24. 260×2=?
25. 1700×5=?
26. 160×30=?
27. 300×48=?
28. 130×60=?
29. 2800×3=?
30. 470×2=?

31. 1300×40=?
32. 380×20=?
33. 1700×30=?
34. 140×300=?
35. 180×40=?
36. 120×12=?
37. 160×50=?
38. 500×19=?
39. 700×130=?
40. 29×30=?
41. 1314×3=?
42. 1912×5=?
43. 1713×4=?
44. 2648×2=?
45. 1815×5=?
46. 918×3=?
47. 514×6=?
48. 1918×4=?
49 2946×2=?
50. 1525×3=?
51. 1327×3=?
52. 2319×4=?
53. 1131×3=?
54. 1618×5=?
55. 2114×3=?
56. 1729×30=?
57. 3213×3=?
58. 1319×50=?
59. 4728×20=?
60. 16140×6=?
61. $1.90×5=?
62. $18.25×3=?
63. $16.40×2=?
64. $13.18×4=?
65. $12.29×3=?
66. $6.48×2=?
67. $7.15×5=?
68. $28.17×3=?
69. $47.25×2=?
70. $15.17×5=?
71. $14.18×4=?
72. $26.45×2=?
73. $18.32×3=?
74. $12.16×6=?
75. $8.19 ×5=?
76. $27.16×3=?
77. $19.25×2=?
78. $47.28×2=?
79. $13.25×3=?
80. $15.18×5=?
81. 504×19=?
82. 1700×40=?
83. $1316×50=?
84. 204×23=?
85. 320×30=?
86. 17019×30=?
87. 141517×5=?
88. 12018×4=?
89. $190.29×3=?
90. $240.45×2=?
91. $180.15×5=?
92. $305×16=?
93. $507.03×13=?
94. $306.04×14=?
95. $1825×30=?
96. $13.15×60=?
97. $25016×30=?
98. $140.15×5=?
99. $1700×40=?
100. $181619×4=?

WRITTEN EXERCISE.

Multiply 4678×537.

4678
537
32746
14034
23390
2512086

Beginning with units, multiply successively by the figures of the multiplier, writing the several products so that figures of the same order may stand in the same column. The several products are then added, the result being the final product.

7. Multiply 7464 by 29.
8. Multiply 9348 by 58.
9. Multiply 7468 by 89.
10. Multiply 567 by 246.
11. Multiply 946 by 708.
12. Multiply 1234 by 567.
13. Multiply $87.46 by 987.
14. Multiply $679.43 by 369.

Find the required products for the following continued multiplications:

15. $64 \times 87 \times 9 = ?$
16. $9 \times 8 \times 6 \times 7 \times 15 = ?$
17. $47 \times 102 \times 64 = ?$
18. $3 \times 9 \times 6 \times 54 \times 42 = ?$
19. $35 \times 127 \times 86 = ?$
20. $53 \times 12 \times 43 \times 124 = ?$

CONTRACTIONS IN MULTIPLICATION.

A contraction is a shorter method or process for obtaining a result than the one ordinarily used.

A number of contractions are employed in Multiplication, the following cases being among the most useful:

CASE I.—Where there are ciphers at the Right of the Multiplier or the Multiplicand. If either factor is 10, 100, 1000, 10000, etc., the product is obtained by writing the ciphers at the right of the other factor. Thus, $10 \times 56 = 560$; $157 \times 1000 = 157000$.

If either factor consists of other figures than 1, with ciphers to the right, multiply regardless of the ciphers, and annex them to the product, thus:

$$\begin{array}{r}
562\\
2400\\
\hline
2248\\
1124\\
\hline
1348800
\end{array}
\qquad\qquad
\begin{array}{r}
234000\\
16\\
\hline
1404\\
234\\
\hline
3744000
\end{array}$$

If both factors have ciphers at the right, multiply together the remaining digits, and annex as many ciphers as there are at the right of both factors, thus:

$$\begin{array}{r}
243000\\
2500\\
\hline
1215\\
486\\
\hline
607500000
\end{array}$$

WRITTEN EXERCISE.

21. Multiply 768 by 1000.
22. Multiply 346 by 4600.
23. Multiply 1,234,000 by 75.
24. Multiply 346,700 by 9000.
25. Multiply 106,000 by 30,400.
26. Multiply 740,000 by 28,000.

CASE II.—When the Multiplier is Composed of Factors.

Multiply 567 by 48.

$$\begin{array}{r} 567 \\ 6 \\ \hline 3402 \\ 8 \\ \hline 27216 \end{array}$$

Since the multiplier equals 6×8, we may employ the continued multiplication 567×6×8, instead of multiplying by 48, thus avoiding the necessity of adding partial products.

WRITTEN EXERCISE.

In the same way, find the required products for the following multiplications, using the factors indicated:

27. $2587\times24(=3\times8)=?$
28. $9376\times63(=7\times9)=?$
29. $75296\times36(=12\times3)=?$
30. $\$875.75\times72(=8\times9)=?$

Find the following products, using factors of the multiplier:

31. $6741\times18.$
32. $54967\times36.$
33. $82973\times48.$
34. $\$458.95\times56.$

CASE III.—When one Part of the Multiplier is a Multiple of Another Part.

Multiply 1254 by 637.

$$\begin{array}{r} 1254 \\ 637 \\ \hline 8778 \\ 79002 \\ \hline 798798 \end{array}$$

Since 63 is 9 times 7, we multiply the first partial product by 9 instead of continuing the multiplication in the usual way.

Multiply 2351 by 856.

$$\begin{array}{r} 2351 \\ 856 \\ \hline 18808 \\ 131656 \\ \hline 2012456 \end{array}$$

Since 56 is 7 times 8, we first multiply by 8, then multiply this partial product by 7.

Multiply 120304 by 48246.

$$\begin{array}{r} 120304 \\ 48246 \\ \hline 721824 \\ 2887296 \\ 5774592 \\ \hline 5804186784 \end{array}$$

Since 24 is 4 times 6, we first multiply by 6, and this product by 4, for the second partial product. Since 48 is 2 times 24, we multiply the second partial product by 2. The sum of the three partial products is the complete product. We could also find the third partial product by multiplying the first one by 8.

WRITTEN EXERCISE.

Perform the following multiplications, shortening the work, wherever practical, by means of any of the foregoing contractions:

35. 3564 × 287 = ?
36. 1356 × 4900 = ?
37. 2456 × 2408 = ?
38. 27800 × 540 = ?
39. 124760 × 56287 = ?
40. 345200 × 7208 = ?
41. 796500 × 24000 = ?
42. 324610 × 9072 = ?

WRITTEN PROBLEMS.

43. Find the value of 267 tons of coal at $4.75 per ton.

44. If there are 16 lots to the block, how many lots are there in the Adams Addition to the town of Mayfair, which extends 7 blocks in one direction and 5 blocks in the other? What is the total value of the Addition, if the average value of the lots is $175 each.

45. On a Dakota ranch there are 2180 sheep. If the fleeces average 8 pounds each, what is the value of the wool clip when the price of the wool is 18 cents a pound?

46. In a certain cornfield there are 469 rows, and 528 hills in each row; how many hills of corn in the field?

47. In the town of Belleville, each of the 6 street cars makes 32 trips per day, and the receipts of each car per trip average $1.45. What are the total receipts for one year of 365 days?

48. When the price of lumber is $18.75 per thousand feet, what is the value of 187 thousand feet?

49. There are 320 rods in one mile; how many rods in 129 miles?

50. A pile of iron rails weighing 65 lbs. to the yard contains 148 rails, each 8 yards long; what is the weight of the pile?

51. The net cost of making a certain book was found to be 59 cents per copy. What was the cost of an edition of 7500 copies?

52. Scott & Co., contractors, employ 75 men at wages averaging $1.75 per day. What is the amount of the pay-roll for one month of 26 working days?

PROMISCUOUS WRITTEN PROBLEMS.

53. Mr. Bennet bought three tracks of land, including respectively, 325, 79, and 146 acres, at $17.50 per acre. He afterwards sold the larger tract at $20 per acre, and the other tracts at $16 25 per acre. Did he lose or gain on the speculation, and how much?

54. John Clark sold to Carter Bros. 79 bushels of potatoes at 45 cents per bu. and 125 lbs. of butter at 17 cents per lb. He received in payment groceries amounting to $17.25, and $20 in cash. How much do Carter Bros. still owe him?

55. Clarence earns $9.50 a week. He estimates his weekly expenses as follows: Board, $3.50; laundry, 40 cents; books and newspapers, 50 cents; incidentals, 75 cents. Allowing $50 per year for clothing, what should he save in one year (52 weeks)?

56. At 35 cents a rod, what will it cost to plant a hedge around a field 65 rods long and 48 rods wide?

57. A lumber dealer sold to a farmer 34 thousand feet of lumber at $16.25 per thousand, receiving in payment 7 tons of hay at $9.50 a ton, 16 cords of wood at $3.75 a cord, and a check for the balance. What was the amount of the check?

58. A grocer bought from a huckster 357 lbs. butter @ 13 cents, and 175 doz. eggs @ 9 cents. He sold the butter @ 16 cents and the eggs @ 12 cents. What was his gain on the purchase?

NOTE.—The character (@) signifies "each" or "apiece," and is used to indicate the market price per pound, per bushel, per doz., etc.

59. A dealer gained $75.90 on 55 tons of baled hay, which he bought at $7.75 per ton. For what sum did he sell it?

60. Miller & Brown, real estate dealers, bought a farm containing 375 acres for $9500. They expended on the property for repairs and buildings, $758.25, and for taxes $102.15. They sold wood from the land to the amount of $95.25, and then sold the entire property for cash at $31.50 per acre. What did they gain through the speculation?

61. A contractor engaged to build a cement walk 155 feet long and 6 feet wide, for $95. He then sublet the work at 9¢ per square foot. How much did he gain on the contract?

DIVISION.

Walter having 12 plums gives 2 to Henry; how many remain? He then gives 2 to Frank, and 2 to Chester; how many are left? How many times can he give 2 away? In the same way successively take 3 out of 15 until nothing is left. How many subtractions are required? How many times can 4 be taken out of 20? Out of 12? Out of 28? How many 5's in 35? In 45?

How many times can you take 3 out of 14? What number will remain?

If Ethel has $25, how many books can she buy at $5 each, and what sum will she have remaining? How often can 9 be taken out of 50, and what number will remain?

A knowledge of what table will enable you to determine the number of 9's in 63, the number of 7's in 42, the number of 8's in 56, etc., without resorting to subtraction?

If 5 pencils cost 30 cents, what do they cost apiece? What would they cost at 1 cent each? How many times is the cost at 1 cent each contained in 30 cents? How much can be paid for each pencil in order to expend the whole of 30 cents?

If a seven-acre lot of land is sold for $56, what is the price per acre? How many times is 2 contained in 10? In 14? In 20?

When we divide a number by 2, we get as a result what part of the number? What is one-half of 16? Of 24? Of 42?

What part of a number do we get when we divide it by three? By 4? By 5? By 12? By 20?

If 5 coats cost $40, 1 coat costs what part of $40? What sum does one coat cost?

Twelve books cost $36, what do they cost apiece?

DEFINITIONS.

Division is the process of finding the number of times one number is contained in another, or of finding one of the equal parts of a number. The first of these processes is termed **Rational Division**; the second, **Partative Division**.

RATIONAL DIVISION.

A **Ratio** is the result obtained by dividing or measuring one number or quantity by another of like kind. Thus, the ratio of $12 to $4 is 3; the ratio of 24 men to 12 men is 2; the ratio of 5 acres to 10 acres is ½, and so on.

In arithmetic, a ratio is indicated by a colon (:) written between the numbers to be compared. Thus, $15:5=3$ indicates that 15 is measured (divided) by 5, and the result is 3. The expression is read, "the ratio of 15 to 5 is 3."

MENTAL EXERCISE.

What is the ratio of $24 to $8? 42 sheep to 6 sheep? 63 days to 9 days? 51 acres to 17 acres? 48 men to 16 men? 6 apples to 2 apples? 2 apples to 6 apples?

Note.—There are always two ways of expressing the ratio, or relative value of two numbers, accordingly as the one or the other is taken as the basis of the comparison: thus, in comparing 12 and 4, we may say "12 is three times 4," or we may say "4 is one-third of 12." In like manner there are two ways of stating a ratio, viz.: "$12:4=3$," or "$4:12=\frac{1}{3}$."

State the value of the following ratios:

$25 : $5; 8 men : 24 men; 32 cents : 2 cents; 100 pounds : 25 pounds; 68 acres : 17 acres; 19 tons : 76 tons; $12 : $108; 78 miles : 26 miles; 91 gallons : 13 gallons; 90 feet : 18 feet.

MENTAL PROBLEMS IN RATIONAL DIVISION.

At 15 cents each, how many pineapples may be bought for 75 cents?

Analysis.—As many pineapples may be bought as 15 cents is contained times in 75 cents, or 5; hence, 5 pineapples may be bought.

Analyze the following:

1. If there are 8 pints in one gallon, how many gallons in 56 pints?

2. How many weeks are there in 84 days?

3. At $3 a case, how many cases of eggs can be bought for $81?

4. If a train is running at the rate of 18 miles an hour, in what time will it run 72 miles?

5. Chester wishes to invest $96 in chickens at $3 a dozen; how many dozen can he buy?

6. At 30 cents each, how many music books can be bought for $6.30?

7. Fred planted 85 hills of potatoes. If there are 17 hills in each row, there were how many rows?

8. If 40 arithmetics may be packed in a box of given size, how many similar boxes will be required for an edition of 6000 books?

9. How many feet in 96 inches?

10. If 20 tons of coal may be loaded on one car how many cars will be required for 780 tons?

11. Of a field containing 80 acres Henry plowed 16 acres; what part of the field did he plow?

ANALYSIS.—Since 16 acres is contained in 80 acres 5 times, he plowed one-fifth of the field.

12. Henry had $84 and has spent $14; what part of the whole amount has he spent?

13. 8 days is what part of 8 weeks?

14. From a bin of wheat containing 560 bushels 140 bushels were sold; what part of the wheat was sold?

15. Arthur has 96 cents; how many pencils can he buy at 4 cents each? After he has purchased 6 pencils, what part of the remainder of his money will be required to purchase two more?

PARTATIVE DIVISION.

When a number or quantity is divided into two equal parts, the parts are called halves. If it is divided into three equal parts, the parts are called thirds; if in four equal parts, fourths, and so on.

The division of a number into equal parts is expressed by writing it above a horizontal line and the number by which it is divided below the line. Thus $\frac{12}{4}$ indicates that 12 is to be divided into four equal parts. The expression is read, "12 fourths," and its value is 3.

MENTAL EXERCISE.

Give the value of the following:

$\frac{21}{7}$; $\frac{42}{6}$; $\frac{51}{17}$; $\frac{24}{6}$; $\frac{50}{10}$; $\frac{63}{9}$; $\frac{96}{16}$; $\frac{72}{12}$; $\frac{85}{17}$; $\frac{65}{13}$; $\frac{64}{16}$; $\frac{81}{3}$; $\frac{76}{4}$; $\frac{91}{7}$.

What is one-fifth of 90? One-twelfth of 72? One-ninth of 81? One-fifteenth of 60? One-thirteenth of 78? One-twenty-ninth of 58? One-nineteenth of 95? One-seventh of 84?

MENTAL PROBLEMS IN PARTATIVE DIVISION.

Frank sold 6 bushels of apples for $9; what was the price per bushel?

Analysis.—Each bushel was sold for one-sixth of $9, or $1.50.

Analyze the following:

1. Henry earned $45 in 15 days; what did he earn in a day?

2. A train ran 87 miles in 3 hours; what was the rate of speed?

3. Fred planted 85 hills of potatoes. If there were 5 rows, there were how many hills in each row?

4. If seven hogs weighed 840 pounds, what was their average weight?

5. Carl raised 150 bushels of potatoes on a six-acre lot; what was the yield per acre?

6. A carload of coal containing 13 tons was sold for $52; what was the price per ton?

7. If there are 18 rows in an orchard containing 720 trees, there are how many trees in each row?

8. A company of men packed 750 tons of ice in 15 days; what was the average quantity packed per day?

9. $108 was paid for a cement walk 2 yards wide and 60 yards long; what was the price per square yard?

10. To fence a lot 20 rods long and 15 rods wide, costs $84; what was the cost of the fence per rod?

DIVISION IN GENERAL.

The **Dividend** is the number to be divided.

The **Divisor** is the number by which we divide.

The **Quotient** is the number which expresses the number of times the Divisor is contained in the Dividend, or the value of the equal parts into which the Dividend is divided.

The **Remainder** is the undivided portion of the Dividend.

Explanation.—When we divide 47 by 8, 47 is the *dividend*, 8 is the *divisor*, 5 is the *quotient*, and 7 is the *remainder*.

METHODS OF INDICATING DIVISION.

The General Sign of Division (÷) is a combination of the colon (:), which indicates Rational Division, and the horizontal line (—), which indicates Partative Division.

When written between two numbers the General Sign indicates that the number to the left of the sign is to be divided by the number to the right. Thus, $24 \div 6 = 4$.

The General Sign of Division may be used to indicate either Rational or Partative Division; or the general process of division irrespective of its application.

Division is also indicated by writing the divisor to the left of the dividend with a curved line between them, the quotient being written below the dividend, thus:

$$\begin{array}{r} 7)\underline{21} \\ 3 \end{array}$$

Or in Long Division, to the right of the dividend, thus:

$$\begin{array}{l} 7)21(3 \\ \underline{21} \end{array}$$

NOTE.—In Algebra, and in some works on arithmetic, the divisor is written to the *right* of the dividend, and the quotient beneath the divisor, thus:

$$\begin{array}{l} 21(7 \\ 21\ \overline{3} \end{array}$$

PRINCIPLES.

1. A ratio can exist only between like numbers.

2. In Rational Division, the quotient is an abstract number.

3. In Partative Division, the divisor is an abstract number, and the quotient is of the same kind as the dividend.

4. In any division, the dividend is equal to the product of the divisor and quotient, plus the remainder, if there be one.

RELATION OF DIVIDEND, DIVISOR, AND QUOTIENT.

A change in the dividend alone produces a like change in the quotient; a change in the divisor alone produces an opposite change in the quotient; the same change in both dividend and divisor does not affect the value of the quotient.

EXPLANATION.—If the dividend alone be multiplied or divided, the quotient will be correspondingly multiplied or divided; but if the divisor alone be multiplied, the quotient is divided; or, if the divisor be divided, the quotient is multiplied. If both dividend and divisor are either multiplied or divided by the *same number*, the quotient remains the same.

The foregoing law has many important applications in arithmetic, and should be thoroughly understood.

DIVISION PROCESSES.

There are two forms of the division process, called, respectively, *Long Division* and *Short Division*.

Long Division is the full, or complete, form of the division, all the steps or processes of the division being recorded.

Example.

$$\begin{array}{l}
25521 \div 7 = ? \\
7)25521(3645 \\
\underline{21} \\
\;45 \\
\;\underline{42} \\
\;\;32 \\
\;\;\underline{28} \\
\;\;\;41 \\
\;\;\;\underline{35} \\
\;\;\;\;6
\end{array}$$

Explanation.—Taking enough of the left hand digits of the dividend to contain the divisor, we find that 25 (thousands) contains 7, 3 (thousands) times. 3 (thousands) times 7 gives 21 (thousands), which, subtracted from 25 (thousands), leaves 4 (thousands). Annexing the next figure and dividing, we find that 7 is contained in 45 (hundreds) 6 (hundreds) times. Multiplying and subtracting as before, we have left the remainder, 3 (hundreds). Bringing down the next figure of the dividend, we have 32 (tens), which, divided by 7, gives us 4 (tens). Multiplying, subtracting, and bringing down the next figure, as before, we have 41, which, divided by 7, equals 5. Again multiplying and subtracting, we have a final remainder of 6.

Short Division is merely an abbreviated form in which the various multiplications and subtractions are performed mentally, thus:

$$\begin{array}{r}
7)\underline{25521\phantom{+6 \text{ rem.}}} \\
3645+6 \text{ rem.}
\end{array}$$

Explanation.—Ascertaining mentally that 7 is contained in 25, 3 times, with 4 remaining, we write the 3 as the first figure of the quotient, and *mentally prefix* the 4 to the next figure, which gives 45 as the dividend in the next step, and so on.

Short Division can be employed only when the divisor is a number small enough for us to perform the multiplications and subtractions of the process mentally. The student should never employ Long Division unless the divisor exceeds 12.

MENTAL EXERCISE.

Give the results at sight of the divisions here indicated. Read the results from top to bottom, and then from left to right:

(*1*)	(*2*)	(*3*)	(*4*)	(*5*)	(*6*)	(*7*)	(*8*)
2)26	2)52	2)72	4)84	13)26	13)39	31)62	15)75
3)45	2)34	5)75	7)91	14)42	13)78	14)70	15)90
2)38	3)54	2)74	5)85	29)58	17)68	24)72	31)93
3)39	2)56	2)78	3)87	32)64	23)69	29)87	19)95
2)54	4)60	4)76	6)84	18)36	18)90	27)81	14)98
2)28	3)57	5)85	5)90	17)51	18)54	16)64	47)94
3)42	5)65	3)81	2)92	13)65	19)57	6)90	16)80
2)46	2)64	2)82	3)93	17)34	18)72	23)92	22)88
4)52	4)68	2)84	2)94	16)48	26)52	14)42	36)72
2)32	3)69	4)72	4)96	14)56	21)63	13)91	32)96
2)58	3)63	5)90	6)96	19)38	17)85	26)78	39)78
3)51	5)70	6)78	7)98	21)84	15)60	24)96	13)52

Give the quotients and remainders of the following:

(*9*)	(*10*)	(*11*)	(*12*)	(*13*)	(*14*)	(*15*)	(*16*)
7)24	8)44	8)41	6)71	7)55	26)53	13)55	15)100
5)36	7)62	7)60	12)59	12)82	15)76	14)79	21)100
8)42	6)43	6)51	11)60	19)97	13)68	18)91	12)1000
9)56	5)52	12)62	9)89	3)53	14)58	14)100	18)100
5)38	9)49	8)20	6)70	18)59	24)100	16)100	17)100

WRITTEN PRACTICE.

1. Divide 1978 by 7.
2. Divide 8976 by 6.
3. Divide 10274 by 8.
4. Divide 7861247 by 4.
5. Divide 20761201 by 5.
6. Divide 12172946 by 9.
7. Divide $8673.57 by 3.
8. Divide $407232.45 by 9.
9. Divide 9706421 by 11.
10. Divide 201634596 by 12.
11. 21786÷19=?
12. 87463÷28=?
13. 10271086÷146=?
14. 94207658÷67=?
15. 560217563÷496=?
16. 85205617÷649=?
17. 417601924÷4567=?
18. 93681596÷837=?
19. 170215862÷299=?
20. 37021675÷7019=?

CONTRACTIONS IN DIVISION.

CASE I.—Where the Divisor is 10, 100, 1000, etc.

56743 ÷ 1000=56.743. Where the divisor consists of the digit, 1, with ciphers annexed, the result is obtained by merely pointing off from the left of the dividend as many figures as there are ciphers in the divisor. The figures thus pointed off comprise the remainder; the quotient consists of the remaining figures. Thus, in the accompanying example, the quotient is 56, and the remainder is 743.

MENTAL EXERCISE.

Give at sight the quotients and remainders for the following divisions:

1. 124763÷100.
2. 96538÷10.
3. 2016043÷1000.
4. 4873620÷10000.
5. 1862746÷1000.
6. 9021638÷100000.
7. 4267358602÷1,000,000.

CASE II.—Where the Divisor consists of other digits than 1 with Ciphers at the Right.

```
246713 ÷ 1700 = ?
 17|00)2467|13(145
       17
        76
        68
         87
         85
          213
```

Cut off the ciphers of the divisor, and from the right of the dividend, an equal number of figures. Divide the remaining figures of the dividend by the divisor, exclusive of its ciphers. To the first remainder, if there be one, annex the figures cut off from the right of the dividend, for a complete remainder.

WRITTEN EXERCISE.

21. 24678÷1400=?
22. 18276201÷9000=?
23. 74216730÷870000=?
24. 8726013÷52000=?
25. 674120167÷24600=?

PROBLEMS FOR MENTAL ANALYSIS.

Note—In analyzing these problems, the student should carefully distinguish between those involving Rational and those involving Partative Division.

1. A train moving at the rate of 17 miles per hour will require how long to run 85 miles?

2. A lady received 3 pounds of coffee for 7 doz. eggs @ 9c. What was the coffee worth per pound?

3. Four men built 60 rods of fence in 3 days. What was the average number of rods per day for each man?

4. If cranberries are worth 15c. a quart, how many quarts should be given in exchange for 5 pounds of butter @ 18c.?

5. A torpedo boat running at the rate of 24 knots an hour, will steam how far in 15 minutes?

6. A grocer sells lemons at 20c. a dozen and oranges at 30c. At these prices, how many oranges are worth the same as 36 lemons?

7. Henry and his 5 brothers share equally a legacy of $18000. Henry invests his share in land at $15 an acre. How many acres does he purchase?

8. Five men united in a speculation, buying 5000 bushels of grain @ 65c. and selling it at 75c. If the expenses were $100 and they shared the profits equally, what sum did each gain?

9. In a certain town the water tax is 13 cents per thousand gallons, with 15 cents per quarter for meter rent. If Mr. A's quarterly water bill is $2.75, how much water was used?

10. Five heirs inherit an estate valued at $60,000, which they are to share equally. Afterwards another heir is found; to what amount will this discovery diminish the share of each of the other five?

WRITTEN PROBLEMS.

26. It requires about 8 minutes for light to pass from the sun to the earth, a distance of 92,500,000 miles. What is the approximate velocity of light per minute? What is the velocity per second?

27. During 18 days in the month of May, the total attendance at the Fairview school amounted to 576 days. What was the average daily attendance? If there are 36 pupils enrolled, what was the average number of days each pupil attended during the month?

28. A tract of land containing 2529 acres was sold for $42360.75; what was the price per acre?

29. By a drop of 7 cents in the market price of wheat, a speculator lost $1330; how many bushels did he buy?

30. The product of two numbers is 40548. One of the numbers is 327; what is the other?

31. Mr Howland sold his spring wool clip at 17 cents a pound, receiving $400.86. How many pounds did he sell?

32. A steam pump with a capacity of 575 gallons a minute, will require how long to fill a tank holding 32775 gallons.

33. By a rise of 25 cents per bushel in the market price, the value of a farmer's potato crop is increased $198.75. How many bushels did he raise?

34 If a cruiser can sustain an average speed of 19 knots an hour, how many days and hours will she require to cover a distance of 5662 knots?

35. Mr. L. finds that he spends for tobacco an average of $2.75 per month. In how many years and months would this expenditure amount to enough to purchase a $500 piano?

COMBINATIONS OF ARITHMETICAL PROCESSES.

It is often desirable to express in one formula, or statement, several arithmetical operations; also to indicate the order in which these operations are to be performed.

This is accomplished by means of the four arithmetical signs $+$, $-$. $\times$, and $\div$, used in connection with the comma, the parenthesis ( ), and the vinculum $\overline{\qquad\qquad}$.

Thus, the expression $6+7, \times 9$ indicates that 6 and 7 are to be added, and the sum multiplied by 9, the result being 117; while the expression $6, +7\times 9$ indicates that the product of 7×9, or 63, is to be added to 6, the result being 69.

The parenthesis is used to inclose one or more indicated operations, the result of which is to be considered as one quantity. Thus, $5+(12\div 3)+8-(2\times 3)$ indicates that the quotient of $12\div 3$, or 4, is to be added to 5, that 8 is to be added to this, and that from this result, 17, the product of 2×3, or 6, is to be subtracted, giving the result, 11.

The vinculum may be used instead of, or in connection with, the parenthesis. Thus, $27-\overline{24\div(12-8)}$ indicates that the whole quantity, 24 divided by the quantity $12-8$, is to be subtracted from 27.

NOTE.—Division may also be conveniently indicated by writing the divisor under the dividend with a horizontal line between them. Thus the last formula may be written:

$$27-\frac{24}{12-8}$$

MENTAL EXERCISE.

Verify the following:

1. $(17-8)\div 3=3.$

2. $\overline{51\div 17}+9, -(2\times 4)=4.$

3. $34\div(15+\overline{(3\times 2)\div 3})=2.$

4. $\dfrac{(75\div 15)+9}{2}=7.$

5. $(98-7)\div(\overline{8-4}-\overline{28-9})=7.$

Find the results of the following:

1. $54\div(12+\overline{78\div 13})+15-(8-4)=?$

2. $\dfrac{(25-6)\times(52\div 13)}{(3\times 13)-37.}=?$

3. $(7+8+9,\div 6)\times(56\div\overline{19-5})\times 6=?$

4. $\frac{48}{17+7}\times(\overline{52\div 2},-(4+8,\times 2)\ )=?$

5. $((3\times 12)-24)\times(76\div 19)+15=?$

WRITTEN EXERCISE.

Mr. A. sold some cattle for $950 and some sheep for $375. With a part of the money he bought 3 cows at $35 each and with the remainder bought 8 horses. What was the average cost of the horses?

FORMULA FOR SOLUTION.

$\overline{(\$950+\$375)-(3\times\$35)}\div 8=$ Average cost of horses.

In a like manner indicate the formulas for the solution of the following:

1. A man bought a house and lot for $1500, expended $250 for repairs, and then sold the property for $2000, investing his gains in corn at 25c. a bushel. How many bushels did he buy?

2. A certain tank holds 40,000 gallons. It has a supply pipe that brings in 150 gallons per minute, and a discharge pipe that carries away 175 gallons per minute. If the tank is full, and both pipes are left open, how many hours will be required to empty the tank?

3. A man bought a farm for $5000, 60 head of cattle at $25 each, and 15 horses at $75 each. He then sold the farm and stock for $8000. What sum did he gain?

4. If there are 160 square rods in an acre, what is the value at $25 an acre of a field 96 rods long and 70 rods wide?

5, Mr. Markham earns $2.75 per day and works an average of 5 days each week. His weekly expenses are as follows: Board, $3.75; laundry, 50c.; clothing, $2; other expenses, $1. The remainder of his earnings he deposits in bank. What sum should he have to his credit at the end of one year, or 52 weeks?

6. Mr. L. sold 965 bushels of wheat at 75 cents, receiving in payment 5 milch cows, a town lot valued at $125, 140 stock hogs worth $2.50 each, and cash $26. What was the average cost of the cows?

7. Ellerton is one of five heirs who inherits an estate consisting of 640 acres of land valued at $45 per acre. If Ellerton has other property valued at $2250, cash $500, and owes debts to the amount of $3000, what is his present worth after his debts are paid?

8. A residence property rents for $35 per month. The annual taxes average $75 and the repairs $50. The owner pays the water rate of $1.75 per month; what is the net income from the property during a period of five years?

9. A committee has a certain sum with which to purchase a carpet for a lodge room. They find that if they purchase carpeting at $1.10 per yard, they will lack $15.60 of having enough money to pay for it, but if they purchase a grade at 95 cents per yard, they will have $8.40 of their fund unexpended. How many yards are required?

10. Walter Bright bought a 30-acre wood lot at $26 per acre. He paid $275 for fencing the lot, sold from it 160 cords of wood at $1.50 per cord, and then sold the lot for $950. What did he gain by the speculation?

After writing the formulas required by the foregoing problems find the result of each.

PROMISCUOUS WRITTEN PROBLEMS.

1. A mill owner sold 260,000 shingles at $2.55 a thousand and with the money bought 17 acres of land. What did the land cost an acre?

2. A man paid $80.75 for a cement walk 5 feet wide and 153 feet long. If there are 9 square feet in a square yard, what did the walk cost per square yard?

3. An orchard contains 26 rows of trees, with 42 trees in each row. The apple crop from the orchard was sold for $4095. What was the value of the yield per tree?

4. A real estate dealer sold a tract of 278 acres of land for $12510, receiving $13 per acre more than it cost him. What did he pay per acre for the tract?

5. A farmer sold his wheat crop at 85c. per bushel, and with the money bought 124 stock hogs at $4.25 each. How many bushels of wheat did he sell?

6. A retailer's sales for one week were as follows: Monday, $356.90; Tuesday, $410.50; Wednesday, $387.45; Thursday, $274.15; Friday, $334.93; Saturday, $512.29. What was the amount of the average daily sales?

7. Lillian finds that if she buys a certain number of yards of dress goods at $1.25 a yard, she will have $1.50 remaining. But if she buys the same number of yards of a quality that costs $1.44 a yard, she will lack 78 cents of having enough money to pay for it. How much money has she?

8. A locomotive runs at the rate of 4 rods per second. If there are 320 rods in a mile, how many miles and rods will the locomotive run in an hour.

9. A man bought 16480 pounds of wool @ 19c., and sold it @ 26c. He then invested the gains of this speculation in pork at 8c. a pound, which he afterwards sold at 11c. a pound. What did he gain on the pork?

10. Walter's expense account while attending Marion College from Jan. 1 to June 1, 1900, was as follows: Board and room, $75.50; tuition and books, $62.85; clothing and laundry, $41.15; incidentals, $7.95. What were his average monthly expenses?

11. The items of cost for an edition of 6000 books are as follows: paper, $318.25; printing and binding, $1345.80; packing, $15.95. If the books are sold at 50c. each, what is the gain on each book?

12. A speculator bought a piece of city property for $2650, sold the property at an advance of $206 over the purchase price, and invested the money in wheat at 68c. a bushel. He afterward sold the wheat at 74c. a bushel, paying commission and other charges to the amount of $62.75. How much was his original capital increased?

13. A buyer for a department store is allowed a certain sum with which to purchase bicycles. He bought 55 bicycles at $27 each, and had $15 remaining. Supposing he had bought bicycles at $32 each, how many could he have purchased, and what sum would remain unexpended?

14 Benton & Sons sold 528 bbls. of flour at $3.75 a barrel, and invested the proceeds in pickled pork at $17 a barrel. How many barrels of pork did they buy, and what sum remained unexpended.

15. A contractor engages to build a house for $2875, provided he uses lumber costing $18.50 per thousand feet; but if lumber worth $20 per thousand is used, he will ask $2945.50. If the cheaper grade is selected, what will be the amount of the lumber bill?

16. On a field of wheat containing 75 acres, a farmer harvested 1397 bushels; on a second field of 64 acres, he harvested 1123 bushels; and on a third field of 57 acres, he harvested 1008 bushels. What was the average yield per acre for the whole crop?

17. Five men bought a paper mill for $47500. They operated the mill for one year at a total outlay of $48560.75. During the time they sold paper to the amount of $69744.25. They then sold the mill for $50000. What was the amount of each man's share of the gain?

18. A cyclist rode 4 hours and 20 minutes at the rate of 15 miles an hour. He then rode back over the same road in just 5 hours. At what average rate per hour did he ride on his return trip.

19. Mrs. B finds that if she selects for a new gown material that costs $1.25 per yard, the whole cost of the garment will be $27.85; but if she selects material worth $1.48, the cost will be $30.61. What is the cost of the gown aside from the cloth?

20. By the terms of Mr. Beverly's will, his estate, which is valued at $375000, is to be divided as follows: his wife is to receive $25,000; the eldest son, $20,000; each of the three remaining children, $10,000. After paying these and the other special bequests to the amount of $5750, the remainder of the estate is to be divided equally among the five heirs. What is the value of each one's interest in the undivided portion of the estate?

Progressive Commercial Arithmetic.

PART II.